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Spatial Alignment of Passive Radar and Infrared Image Data on Seeker

Jin Tang^a, Rui Ding^b, a*^a*Department of Automation, Beijing University of Posts and Telecommunications, Beijing, China*^b*Department of Electronic, Capital Normal University, Beijing, China*

Abstract

The line-of-sight (LOS) and LOS rate is essential for proportional navigation. However, on the hybrid seeker with passive radar sensor and infrared image sensor, they can not be obtained directly for they are observed in different observing space. Then, observations obtained from passive radar sensor and infrared image sensor need to be share a consistent index of space, that is spatial alignment. The exact LOS and LOS rate dynamic models of passive radar sensor and infrared image sensor are constructed respectively.

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1. Introduction

Most modern homing missiles make use of PN (proportional navigation) guidance law based on LOS (line of sight) vector and its rate^{[1][2]}. The hybrid seeker system utilizes passive radar sensor and infrared image sensor to get the information of LOS and LOS rate. More precise result can be obtained by fusing data from the two kinds of sensor^[3]. However, the observed signals from passive radar and for the infrared image sensor have different form and norm. To accomplish sensor fusion, it is necessary for different sensors to agree on a common notion of space. That is, observations obtained from different sensors need to be share a consistent index of space.

2. LOS and LOS rate from passive radar

The principle of angle measurement from passive radar is that, two or more independent antennae accept the signal from target in the same time, and then compare the measurements to get the relative

* Corresponding author. Tel.: 010-62283022.

E-mail address: tangjin@bupt.edu.cn.

angle information of the target. This angle measurement value is the vector relative the place where the antennae are located, while the LOS is relative to the missile body coordinate. Therefore, the angle measurement need transformed to LOS, and then LOS rate is acquired for the proportional navigation^[4].

First of all, the body coordinate frame $OX_B Y_B Z_B$ is built, which is glued on the missile body. Define a right-handed coordinate system as showed in the figure 1, where OX_B always points to the missile nose direction along the roll axes, OY_B is the axis of pitch, and OZ_B is the axis of row which completes the mutually orthogonal right-handed coordinate system. Providing that there is an inherent angle θ between the signal reference plan of passive radar and the body coordinate frame, which is usually decided by the mechanical position of antennae, then the radar signal coordinate frame $OX_R Y_R Z_R$ can be built. One way to describe the rotation (the change in orientation) of a rigid body is by means of Euler angles. Then, the radar signal coordinate frame can be described with that OX_R has same direction with OX_B , while OY_R and OZ_R can be regard as the rotated OY_B and OZ_B by angle $-\theta$ (the rotation directions are represented by the right hand rule).

Then, the coordinate-transformation matrix from the radar signal coordinate frame $OX_R Y_R Z_R$ to the body coordinate frame $OX_B Y_B Z_B$ is the following:

$$\mathbf{L}_{BR} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} \quad (1)$$

It is quit convenient to get the position vector $(x_b, y_b, z_b)^T$ in the body coordinate frame from the target position vector $(x_r, y_r, z_r)^T$ in the radar signal coordinate frame through the formula:

$$(x_b, y_b, z_b)^T = \mathbf{L}_{BR}(x_r, y_r, z_r)^T \quad (2)$$

However, through passive radar, the relative distance between the missile and its target is unobservable, and the only the angle bias vector $(\varepsilon_{ry}, \varepsilon_{rz})^T$ is measurable. the angle bias vector $(\varepsilon_{ry}, \varepsilon_{rz})^T$ means that the target direction can be reached by twice successive rotation from the radar coordinate frame $OX_R Y_R Z_R$: first, rotate $OX_R Y_R Z_R$ around the axes OZ_R with angle ε_{rz} , and then rotate the new "OX_R Y_R Z_R" around it new axes "OY_R" with angle ε_{ry} to acquire the target coordinate frame $OX_P Y_P Z_P$ (the rotation directions are represented by the right hand rule), and the angle has relation to location vector $(x_r, y_r, z_r)^T$, and comply to:

$$\tan \varepsilon_{rz} = y_r / x_r, \tan \varepsilon_{ry} = -z_r / \sqrt{x_r^2 + y_r^2} \quad (3)$$

Substitution of (3) in (2) lead to:

$$\begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x_r \\ \tan \varepsilon_{rz} x_r \\ -\tan \varepsilon_{ry} \sqrt{1 + \tan^2 \varepsilon_{rz}} x_r \end{pmatrix} = \begin{pmatrix} 1 \\ \cos \theta \tan \varepsilon_{rz} + \sin \theta \tan \varepsilon_{ry} \sqrt{1 + \tan^2 \varepsilon_{rz}} \\ \sin \theta \tan \varepsilon_{rz} - \cos \theta \tan \varepsilon_{ry} \sqrt{1 + \tan^2 \varepsilon_{rz}} \end{pmatrix} x_r = \begin{pmatrix} 1 \\ A \\ B \end{pmatrix} x_r \quad (4)$$

The line of sight (LOS) between missile and target in the body coordinate frame is defined the same way as that in the radar signal coordinate frame, which is:

$$\tan \varepsilon_{bz} = y_b / x_b, \tan \varepsilon_{by} = -z_b / \sqrt{x_b^2 + y_b^2} \quad (5)$$

Then from formula (4) (5), the line of sight between missile and target is:

$$LOS_{radar} = \begin{pmatrix} \varepsilon_{by} \\ \varepsilon_{bz} \end{pmatrix} = \begin{pmatrix} \arctan(A) \\ \arctan(-B / \sqrt{1 + A^2}) \end{pmatrix} \quad (6)$$

Since the coordinate frame $OX_P Y_P Z_P$ is successively rotated from the radar coordinate frame $OX_R Y_R Z_R$ through the Euler angle $(\varepsilon_{ry}, \varepsilon_{rz})^T$, then the coordinate-transformation matrix L_{PR} is:

$$L_{PR} = R(\varepsilon_{ry})R(\varepsilon_{rz}) = \begin{pmatrix} \cos \varepsilon_{ry} & 0 & \sin \varepsilon_{ry} \\ 0 & 1 & 0 \\ -\sin \varepsilon_{ry} & 0 & \cos \varepsilon_{ry} \end{pmatrix} \begin{pmatrix} \cos \varepsilon_{rz} & -\sin \varepsilon_{rz} & 0 \\ \sin \varepsilon_{rz} & \cos \varepsilon_{rz} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (7)$$

Then with corresponding reverse rotate, the radar coordinate frame $OX_R Y_R Z_R$ can also be transformed from $OX_P Y_P Z_P$ with matrix L_{RP} : $L_{RP} = R^{-1}(\varepsilon_{rz})R^{-1}(\varepsilon_{ry})$. By the basic Euler angle equation^[5], the relation between the Euler angle velocity and of the angular velocity vector

$$\text{is : } \omega_{RP}^R = \begin{pmatrix} \dot{\omega}_{rp}^x \\ \dot{\omega}_{rp}^y \\ \dot{\omega}_{rp}^z \end{pmatrix} = R^{-1}(\varepsilon_{rz}) \begin{pmatrix} 0 \\ \dot{\varepsilon}_{ry} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\varepsilon}_{rz} \end{pmatrix} = \begin{pmatrix} (\sin \varepsilon_{rz})\dot{\varepsilon}_{ry} \\ (\cos \varepsilon_{rz})\dot{\varepsilon}_{ry} \\ \dot{\varepsilon}_{rz} \end{pmatrix}, \omega_{BP}^B = \begin{pmatrix} \dot{\omega}_{bp}^x \\ \dot{\omega}_{bp}^y \\ \dot{\omega}_{bp}^z \end{pmatrix} = R^{-1}(\varepsilon_{bz}) \begin{pmatrix} 0 \\ \dot{\varepsilon}_{by} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\varepsilon}_{bz} \end{pmatrix} = \begin{pmatrix} (\sin \varepsilon_{bz})\dot{\varepsilon}_{by} \\ (\cos \varepsilon_{bz})\dot{\varepsilon}_{by} \\ \dot{\varepsilon}_{bz} \end{pmatrix}, \text{Where}$$

ω_{RP}^R represents the angular velocity vector of the target coordinate relative to the radar coordinate, and ω_{BP}^B represents the angular velocity vector of the target coordinate relative to the missile body coordinate. From (2), the equation between them is: $\omega_{BP} = L_{BR}\omega_{RP}$.

Finally, the LOS rate can be got:

$$LOS_{radar} = \begin{pmatrix} \dot{\varepsilon}_{by} \\ \dot{\varepsilon}_{bz} \end{pmatrix} = \begin{pmatrix} (\sin \varepsilon_{rz} / \sin \varepsilon_{bz})\dot{\varepsilon}_{ry} \\ (\sin \theta \cos \varepsilon_{rz})\dot{\varepsilon}_{ry} + (\cos \theta)\dot{\varepsilon}_{rz} \end{pmatrix} \quad (8)$$

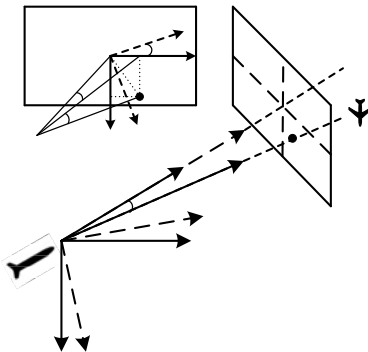


Figure 1. Geometry of passive radar

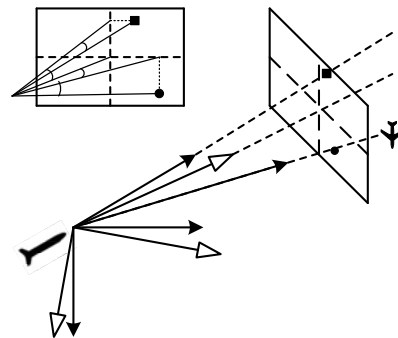


Figure 2. Geometry of infrared image sensor

3. LOS and LOS rate from infrared image

As the infrared image sensor is mounted on the two-degree-of-freedom (2-DOF) gimbals, more parameters are need for the LOS and LOS rate^{[6][7]}. The body coordinate frame $OX_B Y_B Z_B$ is built same as that in section 2; the gimbals' coordinate frame $OX_G Y_G Z_G$, whose axes OX_G points to the direction same with gimbals, can be reached by twice successive rotation from the body coordinate frame $OX_B Y_B Z_B$: first, rotate $OX_B Y_B Z_B$ around the axes OZ_B with angle η , and then rotate the new " $OX_B Y_B Z_B$ " around it new axes " OY_B " with angle λ (the rotation directions are represented by the right hand rule). The infrared image sensor is fixed with the gimbals and the optical axis is regarded the same as the gimbals' axis OX_G for convenience. By using sensor information of the target position, the gimbals are controlled to point to the target. That is the gimbals' axis OX_G always approaches the target direction OX_P . However, Even if the target is tracked by sensor pointing control, it is not necessarily on the optical axis^[8]. The target and the seeker vehicle platform (i.e., the body) may be maneuvering and the sensor stabilization and pointing cannot be made perfect. Consequently, the target position may be somewhere off the optical axis, as illustrated in figure 2.

Y_R

θ

Y_B

P

ε_{bz}

A new coordinate system $OX_P Y_P Z_P$ is introduced in such a way that the target is on the OX_P axis. This system is further defined by two Euler rotations of the gimbals' system to the target: first ε_{gz} (yaw) about the axis OZ_G , the ε_{gy} (pitch) about the subsequent axis " OY_G ". The target is then on the axis " OX_G ", a condition that determines the size of the rotations. Since the OX_P axis is pointing to the target permanently, we call this system the target coordinate frame. The target angular position $(\varepsilon_{gz}, \varepsilon_{gy})$ is measured by the sensor. For an IR sensor this is accomplished by image processing in combination with a detection process. The angular position is then determined from the position of a certain detector element in the focal plane.

With the Euler rotate angle (λ, η) , the coordinate-transformation matrix from the gimbals' coordinate frame $OX_B Y_B Z_B$ to the body coordinate frame $OX_G Y_G Z_G$ is the following:

$$\mathbf{L}_{GB} = \mathbf{R}(\lambda)\mathbf{R}(\eta) = \begin{pmatrix} \cos\lambda & 0 & \sin\lambda \\ 0 & 1 & 0 \\ -\sin\lambda & 0 & \cos\lambda \end{pmatrix} \begin{pmatrix} \cos\eta & -\sin\eta & 0 \\ \sin\eta & \cos\eta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (9)$$

The coordinate-transformation matrix from the body coordinate frame $OX_G Y_G Z_G$ to the gimbals' coordinate frame $OX_B Y_B Z_B$ is reverse of (9): $\mathbf{L}_{BG} = \mathbf{R}^{-1}(\eta)\mathbf{R}^{-1}(\lambda)$. If the target location can be defined as $\mathbf{r}_G = (x_g, y_g, z_g)^T$ in the gimbals coordinate frame, while $\mathbf{r}_B = (x_b, y_b, z_b)^T$ in the body coordinate frame, then we can get: $\mathbf{r}_B = \mathbf{L}_{BG}\mathbf{r}_G$. And the target angular $(\varepsilon_{gz}, \varepsilon_{gy})$ observed can be deduced from the target position vector: $\tan \varepsilon_{gz} = y_g / x_g$, $\tan \varepsilon_{gy} = -z_g / \sqrt{x_g^2 + y_g^2}$, and then lead to:

$$\begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = \begin{pmatrix} \cos\eta\cos\lambda & \sin\eta & -\cos\eta\sin\lambda \\ -\sin\eta\cos\lambda & \cos\eta & \sin\eta\sin\lambda \\ \sin\lambda & 0 & \cos\lambda \end{pmatrix} \begin{pmatrix} x_g \\ (\tan\varepsilon_g)x_g \\ -(\tan\varepsilon_g)\sqrt{1+\tan^2\varepsilon_g}x_g \end{pmatrix} = \begin{pmatrix} (\cos\eta\cos\lambda + \sin\eta\tan\varepsilon_g + (\cos\eta\sin\lambda\tan\varepsilon_g)\sqrt{1+\tan^2\varepsilon_g}) \\ (-\sin\eta\cos\lambda + \cos\eta\tan\varepsilon_g - (\sin\eta\sin\lambda\tan\varepsilon_g)\sqrt{1+\tan^2\varepsilon_g}) \\ \sin\lambda - (\cos\lambda\tan\varepsilon_g)\sqrt{1+\tan^2\varepsilon_g} \end{pmatrix} x_g = \begin{pmatrix} C \\ D \\ E \end{pmatrix} x_g \quad (10)$$

The equation between the line of sight (LOS) between missile and target in the body coordinate frame and the position vector is presented in (5). From (5) and (10), we get :

$$LOS_{image} = \begin{pmatrix} \varepsilon_{by} \\ \varepsilon_{bz} \end{pmatrix} = \begin{pmatrix} \arctan(y_b / x_b) \\ \arctan(-z_b / \sqrt{x_b^2 + y_b^2}) \end{pmatrix} = \begin{pmatrix} \arctan(D / C) \\ \arctan(-E / \sqrt{C^2 + D^2}) \end{pmatrix} \quad (11)$$

With the Euler rotate angle $(\varepsilon_{gz}, \varepsilon_{gy})$, the coordinate-transformation matrix between the gimbals' coordinate frame $OX_G Y_G Z_G$ and the body coordinate frame $OX_P Y_P Z_P$ is the following: $\mathbf{L}_{PG} = \mathbf{R}(\varepsilon_{gy})\mathbf{R}(\varepsilon_{gz})$, $\mathbf{L}_{GP} = \mathbf{R}^{-1}(\varepsilon_{gz})\mathbf{R}^{-1}(\varepsilon_{gy})$. From basic kinematics, the following equation holds:

$$\boldsymbol{\omega}_{BP}^B = \boldsymbol{\omega}_{BG}^B + \boldsymbol{\omega}_{GP}^B = \boldsymbol{\omega}_{BG}^B + \mathbf{L}_{BG}\boldsymbol{\omega}_{GP}^G \quad (12)$$

Where $\boldsymbol{\omega}_{BP}^B$ is the angle velocity of target relative to missile body in the body coordinate frame, $\boldsymbol{\omega}_{BG}^B$ is the angle velocity of gimbals relative to missile body in the body coordinate frame, $\boldsymbol{\omega}_{GP}^B$ is the angle velocity of target relative to gimbals in the body coordinate frame. On the other hand, the relation between the Euler angle velocity and of the angular velocity vector can be got from basic Euler angle

$$\text{equation: } \boldsymbol{\omega}_{BP}^B = \begin{pmatrix} (\sin\varepsilon_{bz})\dot{\varepsilon}_{by} \\ (\cos\varepsilon_{bz})\dot{\varepsilon}_{by} \\ \dot{\varepsilon}_{bz} \end{pmatrix}, \boldsymbol{\omega}_{BG}^B = \begin{pmatrix} (\sin\eta)\dot{\lambda} \\ (\cos\eta)\dot{\lambda} \\ \dot{\eta} \end{pmatrix}, \boldsymbol{\omega}_{GP}^G = \begin{pmatrix} (\sin\varepsilon_{gz})\dot{\varepsilon}_{by} \\ (\cos\varepsilon_{gz})\dot{\varepsilon}_{by} \\ \dot{\varepsilon}_{gz} \end{pmatrix}. \quad (13)$$

Substitution of (13) in (12) lead

$$\text{to: } \begin{pmatrix} (\sin\varepsilon_{bz})\dot{\varepsilon}_{by} \\ (\cos\varepsilon_{bz})\dot{\varepsilon}_{by} \\ \dot{\varepsilon}_{bz} \end{pmatrix} = \begin{pmatrix} (\sin\eta)\dot{\lambda} \\ (\cos\eta)\dot{\lambda} \\ \dot{\eta} \end{pmatrix} + \mathbf{L}_{BG} \begin{pmatrix} (\sin\varepsilon_{gz})\dot{\varepsilon}_{by} \\ (\cos\varepsilon_{gz})\dot{\varepsilon}_{by} \\ \dot{\varepsilon}_{gz} \end{pmatrix} = \begin{pmatrix} (\sin\eta)\dot{\lambda} + (\cos\eta\cos\lambda\sin\varepsilon_{gz} + \sin\eta\cos\varepsilon_{gz})\dot{\varepsilon}_{by} - (\cos\eta\sin\lambda)\dot{\varepsilon}_{gz} \\ (\cos\eta)\dot{\lambda} + (-\sin\eta\cos\lambda\sin\varepsilon_{gz} + \cos\eta\cos\varepsilon_{gz})\dot{\varepsilon}_{by} + (\sin\eta\sin\lambda)\dot{\varepsilon}_{gz} \\ \dot{\eta} + (\sin\lambda\sin\varepsilon_{gz})\dot{\varepsilon}_{by} + (\cos\lambda)\dot{\varepsilon}_{gz} \end{pmatrix} = \begin{pmatrix} F \\ G \\ H \end{pmatrix} \quad (14)$$

The LOS rate in the missile body coordinate frame is deduced from (14), and that is:

$$LOS R_{image} = \begin{pmatrix} \hat{\epsilon}_{by} \\ \hat{\epsilon}_{bz} \end{pmatrix} = \begin{pmatrix} G / \cos \epsilon_{by} \\ H \end{pmatrix} \quad (15)$$

4. Conclusion

Since hybrid seeker has passive radar and IR image sensor, the target's angle position could be estimated more precisely with data fusion. However, the observations from two kinds of sensor are different in space and time. Then the exact LOS and LOS rate dynamic models of passive radar sensor and infrared image sensor are constructed respectively. Form (6) and (11), the angle information of target from radar and IR image is transformed to LOS in the missile body coordinate; and form (8) and (15), the angle velocity information of target from radar and IR image is transformed to LOS rate in the missile body coordinate. For application, the simplifying of calculation need more work.

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